Section 8.3

Trig Integral Strategies

In this section, we aim to increase our range of integrals involving trig functions beyond those already contained in our integral tables. The strategies for dealing with products of powers of trigonometric functions will come down to recognizing u-substitutions and utilizing identities (Pythagorean and half-angle) when appropriate.

Pythagorean Identities.

$\sin^2 x + \cos^2 x = 1, \tan^2 x + 1 = \sec^2 x, 1 + \cot^2 x = \csc^2 x$

Half-Angle Identities.

$\cos^2 x = \dfrac{1 + \cos 2x}{2}, \sin^2 x = \dfrac{1 - \cos 2x}{2}$

Strategy for $\int \sin^m x \cos^n x \, dx$.

m odd and positive, n real	Split off one $\sin x$, rewrite the resulting even power of $\sin x$ in terms of $\cos x$, and then substitute $u = \cos x$.
n odd and positive, m real	Split off one $\cos x$, rewrite the resulting even power of $\cos x$ in terms of $\sin x$, and then substitute $u = \sin x$.
m and n both even, nonnegative integers	Use half-angle formulas to transform the integrand into a polynomial in $\cos 2x$, and apply the preceding strategies to the powers of $\cos 2x$ which are greater than 1.

Reduction Formulas for positive integers n

1. $\displaystyle\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$
2. $\displaystyle\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
3. $\displaystyle\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} + \int \tan^{n-2} x \, dx$
4. $\displaystyle\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx, n \neq 1$

Strategy for $\int \tan^m x \sec^n x \, dx$.

n even and positive, m real	Split off a $\sec^2 x$ term, rewrite the remaining even power of $\sec x$ in terms of $\tan x$, and then substitute $u = \tan x$.
m odd and positive, n real	Split off a $\sec x \tan x$, rewrite the remaining even power of $\tan x$ in terms of $\sec x$, and then substitute $u = \sec x$.
m even and positive, n odd and positive	Rewrite the even power of $\tan x$ in terms of $\sec x$ to produce a polynomial in $\sec x$; apply reduction formula 4 to each term.

Example 8.3.1. Evaluate the following integrals.

1. $\displaystyle\int \sin^5 x \, dx$

2. $\displaystyle\int \sin^2 x \cos^2 x \, dx$

3. $\displaystyle\int \sin^3 x \cos^{3/2} x \, dx$

4. $\int \tan x \sec^3 x \, dx$

5. $\int \frac{\sec^4(\ln \theta)}{\theta} d\theta$

6. $\int (1 + \cos 4x)^{3/2} dx$